

Mathematics: analysis and approaches**Standard level****Paper 1**

Name

Date: _____

1 hour 30 minutes

Instructions to candidates

- Write your name in the box above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all of Section A in the spaces provided.
- Section B: answer all of Section B on the answer sheets provided. Write your name on each answer sheet and attach them to this examination paper.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches SL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

exam: 9 pages

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A (36 marks)

Answer **all** questions in the boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 5]

The sum of an infinite geometric sequence is 2. The value of the first term in the sequence is equal to the value of the common ratio r . Find the value of the 3rd term.

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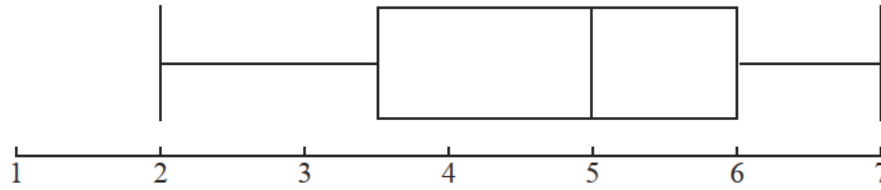
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2. [Maximum mark: 5]

The box and whisker diagram below illustrates the IB grades for a group of 20 students. IB grades are an integer from 1 to 7. The mode grade is 6.



- (a) Write down the median grade. [1]
- (b) Find the number of students who obtained a grade greater than 3. [2]
- (c) Determine, with a reason, the maximum number of students who could obtain a grade of 7. [2]

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3. [Maximum mark: 6]

The angle θ lies in the first quadrant and $\sin \theta = \frac{1}{3}$.

(a) Write down the value of $\cos \theta$. [1]

(b) Find the value of $\cos 2\theta$. [2]

(c) Find the value of $\tan 2\theta$, giving your answer in the form $\frac{a\sqrt{b}}{c}$ where $a, b, c \in \mathbb{Z}^+$. [3]

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4. [Maximum mark: 6]

If $y = x^2 \ln(x)$,

(a) find the x -coordinate of the point M where $\frac{dy}{dx} = 0$; [3]

(b) determine whether M is a maximum or minimum point. [3]

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5. [Maximum mark: 7]



A game consists of a contestant rolling three fair six-sided dice. If a 4, 5 or 6 turns up on any of the three dice, then the contestant loses \$2. If none of the dice turn up a 4, 5 or 6, then the contestant wins \$20.

(a) Show that the contestant expects to win \$3 if the contestant plays the game four times. [4]

One change is made to the game. If none of the dice turn up a 4, 5 or 6, then the contestant wins x dollars.

(b) Find the value of x so that the game is fair. [3]

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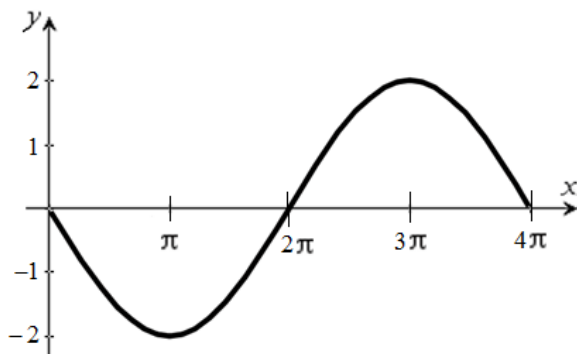
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6. [Maximum mark: 7]

The graph of $f(x) = a \cos[b(x-\pi)]$ for the interval $0 \leq x \leq 4\pi$ is shown below.



- (a) Write down the value of a and the value of b . [2]
- (b) Find the gradient of the graph of f at $x = \frac{3\pi}{2}$. [3]
- (c) Given that $0 \leq c \leq 4\pi$, explain why $\int_c^{4\pi-c} f(x) dx = 0$. [2]

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Do **not** write solutions on this page.

Section B (44 marks)

Answer **all** the questions on the answer sheets provided. Please start each question on a new page.

7. [Maximum mark: 18]

In a class of 85, all of the students must study French or Spanish. Some of the students study both French and Spanish. 51 students study French and 43 students study Spanish.

- (a) (i) Find the number of students who study **both** French and Spanish.
- (ii) Write down the number of students who study **only** Spanish.
- (iii) Write down the number of students who study **only** French. [4]

One student is selected at random from the class.

- (b) Find the probability that the student studies **only** one language. [2]
- (c) Given that the student selected studies **only** one language, find the probability that
- (i) the student studies Spanish;
- (ii) the student studies French. [6]

Let F be the event that a student studies French and S be the event that a student studies Spanish.

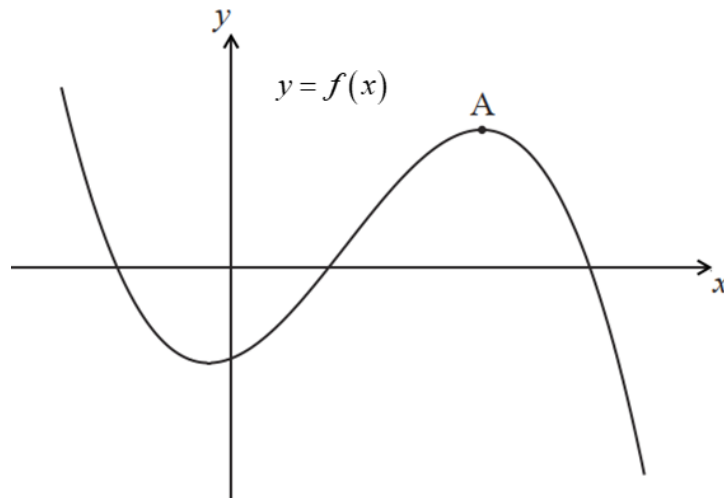
- (d) Determine, with explanation, whether
- (i) F and S are **mutually exclusive** events;
- (ii) F and S are **independent** events. [6]



Do **not** write solutions on this page.

8. [Maximum mark: 14]

The diagram below shows the graph of a function f . There is a local maximum point at A, where $x > 0$.



The derivative of function f is given by $f'(x) = -3x^2 + 8x + 3$.

- (a) Find the x -coordinate of A. [4]
- (b) The graph of function f passes through the point $(1, 0)$. Find an expression for $f(x)$. [5]
- (c) Hence, find the y -coordinate of A. [2]

Consider a new function g such that $g(x) = f(-x) + k$.

- (d) Find the coordinates of the local maximum point on the graph of function g . [3]

9. [Maximum mark: 12]

- (a) Find the value(s) of p such that the equation $4x^2 + px + 1 = 0$ has two equal roots. [3]

The function h is defined as $h(x) = 4\cos x - 4\sin^2 x + 5$, with domain $-360^\circ \leq x \leq 360^\circ$.

- (b) Consider the equation $h(x) = 0$, where $-360^\circ \leq x \leq 360^\circ$.
- (i) State, with a reason, the number of distinct values of $\cos x$ that satisfy this equation.
- (ii) Find all values of x that satisfy this equation. [6]
- (c) Find the range of the function h . [3]

